

Barley Fields Primary School

Calculation Policy



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Sense of Number Maths Consultants

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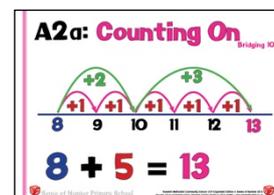
Overview of Calculation Approaches

Early Years into KS1

- Visualisation to secure understanding of the number system, especially the use of place value resources such as, Numicon, 100 Squares and counters.
- Secure understanding of numbers to 10, using resources such as Numicon, Fingers and Multi-link.
- Practical, oral and mental activities to understand calculation.
- Personal methods of recording such as pictorial representations.
- Introduce signs and symbols (**+**, **-** and **=**).

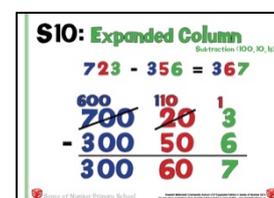
Key Stage 1

- Introduce signs and symbols (**+**, **-**, **x**, **÷**, **=** *in Year 1* and **<**, **>** *signs in Year 2*)
- Extended visualisation to secure understanding of the number system beyond 100, especially of the use of place value resources such as Base 10 (tens and units), Place Value hundred square chart, Number Grids, Arrow Cards and Place Value Counters and numicon.
- Continued use of practical apparatus to support the early teaching of 2-digit calculation. For example, using Base 10 or Numicon to demonstrate partitioning and exchanging before these methods are taught as jottings / number sentences.
- Methods of recording / jottings to support calculation (e.g. partitioning or counting on).
- Use of images such as empty number lines to support mental and informal calculation.



Year 3

- Continued use of practical apparatus, especially Place Value Counters, Base 10 and Numicon to visualise written / column methods before and as they are actually taught as procedures.
- Continued use of mental methods and jottings for 2 and 3 digit calculations.
- Introduction to more efficient informal written methods / jottings including expanded methods and efficient use of number lines (especially for subtraction).
- Column methods, where appropriate, for 3 digit additions and subtractions.



Years 4-6

- Continued use of mental methods for any appropriate calculation up to 6 digits.
- Standard written (compact) / column procedures to be learned for all four operations
- Efficient informal methods (expanded addition and subtraction, grid multiplication, division by chunking) and number lines are still used when appropriate. Develop these to larger numbers and decimals where appropriate.

N.B. Children must be allowed continued access to practical resources to help visualise certain calculations, including those involving decimals.

General Principles of Calculation

When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy.

Whatever method is chosen (in any year group), it must continue to be underpinned by a secure and appropriate knowledge of number facts.

By the end of Year 5, children should:

- have a secure knowledge of number facts and a good understanding of the four operations, in order to:
 - carry out calculations mentally when using one-digit and two-digit numbers
 - use particular strategies with larger numbers when appropriate
- use notes and jottings to record steps and part-answers when using longer mental methods
- **have an efficient, reliable, compact written method of calculation for each operation, which children can apply with confidence when undertaking calculations that they cannot carry out mentally.**

Therefore, when children reach Year 6, they should have all of the relevant strategies in place and be able to use them appropriately within calculations.

Children should always **look at the actual numbers (not the size of the numbers)** before attempting any calculation in order to determine whether or not they need to use a written method.

Therefore, the key question children should always ask themselves before attempting a calculation is: -



The Importance of Vocabulary in Calculation

It is vitally important that children be exposed to the relevant calculation vocabulary throughout their progression through the four operations.

Key Vocabulary: (to be used from Y1)

Addition: Total & Sum Add

E.g. 'The **sum** of 12 and 4 is 16', '12 **add** 4 equals 16'
'12 and 4 have a **total** of 16'

Subtraction: Difference

Subtract (not 'take away' unless the strategy is take away / count back)

E.g. 'The **difference** between 12 and 4 is 8',
'12 **subtract** 4 equals 8'

Multiplication: Product Multiply

E.g. 'The **product** of 12 and 4 is 48',
'12 **multiplied** by 4 equals 48'

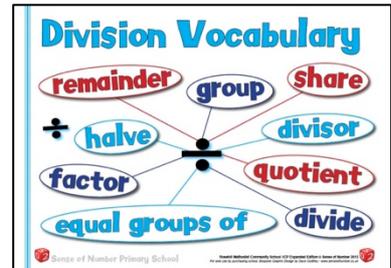
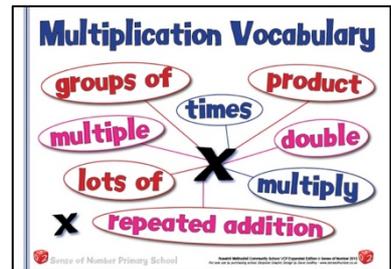
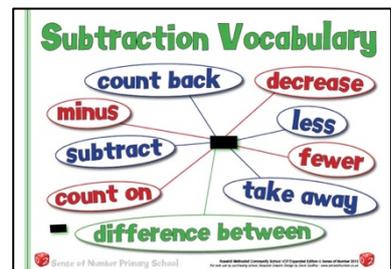
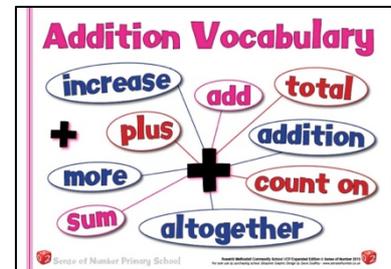
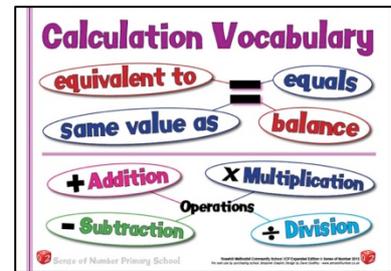
Division: Divisor & Quotient Divide

E.g. 'The **quotient** of 12 and 4 is 3',
'12 **divided** by 4 equals 3'

'When we **divide** 12 by 4, the **divisor** of 4 goes into 12 three times'

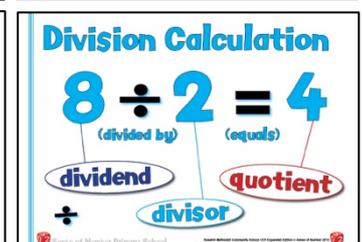
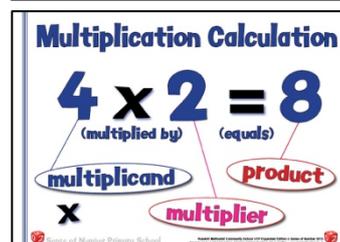
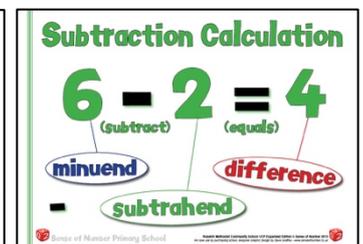
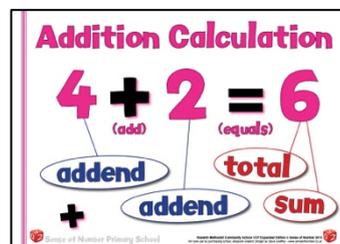
Additional Vocabulary:

The VCP vocabulary posters (below) contain both the key and additional vocabulary children that should be exposed to.



Conceptual Understanding

Using key vocabulary highlights some important conceptual understanding in calculation. For example, the answer in a subtraction calculation is called the difference. Therefore, whether we are counting back (taking away) or counting on to work out a subtraction calculation, we are always finding the difference between two numbers.



Mental Methods of Calculation

Oral and mental work in mathematics is essential, and particularly so in calculation.

Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts.

Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied.

On-going oral and mental work provides practice and consolidation of these ideas. It must give children the opportunity to apply what they have learned to particular cases, exemplifying how the rules and laws work, and to general cases where children make decisions and choices for themselves.

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a **'sense' of number** is the product of structured practice and repetition. It requires an understanding of **number patterns and relationships** developed through **directed enquiry**, of the use of **models and images** and the application of **acquired number knowledge and skills**. Secure mental calculation requires the ability to:

- **Recall key number facts instantly** – for example, all **number bonds to 20**, and **doubles** of all numbers up to **double 20 (Year 2)** and **multiplication facts up to 12×12 (Year 4)**.
- **Use taught strategies to work out the calculation** – for example, by recognising that addition can be done in any order and using this to mentally add a one-digit number to a one-digit or two-digit number (**Year 1**), to add two-digit numbers in different ways (**Year 2**), add and subtract numbers mentally with increasingly large numbers (**Year 5**).
- **Understand how the rules and laws of arithmetic are used and applied** – for example to use **repeated addition** in multiplication (**Year 2**), **estimate** the answer to a calculation and use **inverse operations** to check answers (**Years 3 & 4**), and to use their knowledge of the **order of operations** to carry out calculations involving the four operations (**Year 6**).

The first answer that a child may give to a mental calculation question might be based on instant recall.

E.g. “What is $12 + 4$?”, “What is 12×4 ?”, “What is $12 - 4$?” or “What is $12 \div 4$?” giving the immediate answers “16”, “48”, “8” or “3”

Other children would still work these calculations out mentally by counting on from 12 to 16, counting in 4s to 48, counting back in ones to 8 or counting up in 4s to 12.

From instant recall, children then develop a bank of mental calculation strategies for all four operations, in particular addition and multiplication.

These should be practised regularly until they become refined, where children will then start to see and use them as soon as they are faced with a calculation that can be done mentally.

MA4: Double & Adjust

$$45 + 46 = 91$$
$$45 + 45 + 1$$
$$90 + 1 = 91$$

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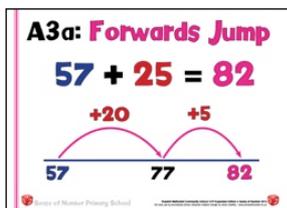
Informal Written Methods and Mental Jottings

The **New Curriculum for Mathematics** sets out the progression of written methods for calculation, which highlights the compact written methods for each of the four operations. It also places emphasis on the need to **'add and subtract numbers mentally'** (Years 2 & 3), mental arithmetic **'with increasingly large numbers'** (Years 4 & 5) and **'mental calculations with mixed operations and large numbers'** (Year 6).

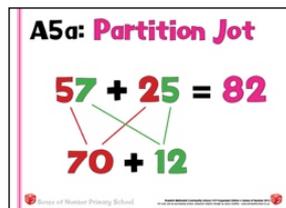
Barley Fields Maths policy (especially in the progression of addition and multiplication) provides very clear guidance not only as to the development of formal written methods, but also the jottings, expanded and informal methods of calculation that embed a sense of number and understanding before column methods are taught. These extremely valuable strategies include:

Addition –

number lines



partitioning



expanded methods



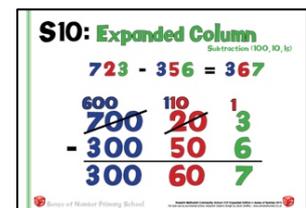
(In addition to the 5 key mental strategies for addition - see 'Addition Progression')

Subtraction –

number lines (especially for counting on)

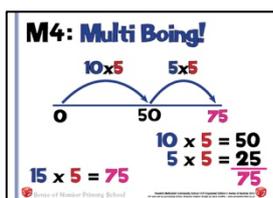


expanded subtraction

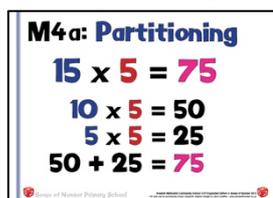


Multiplication –

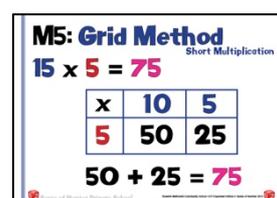
number lines



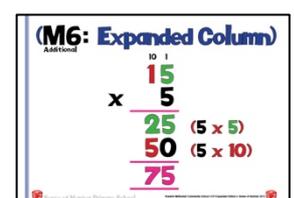
partitioning



grid method



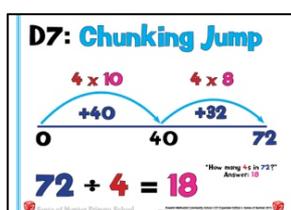
expanded



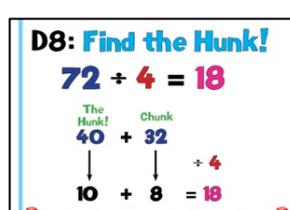
in addition to the key mental strategies for multiplication (see 'Multiplication Progression')

Division –

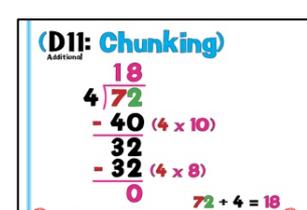
number lines



chunking (as a jotting)



chunking (written method)



Formal (Column) Written Methods of Calculation

The aim is that by the end of **Year 5**, the great majority of children should be able to **use an efficient written method for each operation with confidence and understanding with up to 4 digits**.

This guidance promotes the use of what are commonly known as 'standard' written methods – methods that are efficient and work for any calculation, including those that involve whole numbers or decimals. They are compact and consequently help children to keep track of their recorded steps.

Being able to use these written methods gives children an efficient set of tools, which they can use when they are unable to carry out the calculation in their heads or do not have access to a calculator. We want children to know that they have a reliable, written method to which they can turn when the need arises.

The entitlement to be taught how to use efficient written methods of calculation is set out clearly in the National Curriculum objectives. Children should be equipped to decide when it is best to use a mental or written method, based on the knowledge that they are in control of this choice as they are able to carry out all methods with confidence.

Barley Fields Maths policy recognises that whilst children should be taught the efficient, formal written calculation strategies, **it is vital that they have exposure to models and images, and have a clear conceptual understanding of each operation and each strategy**.

The visual slides that feature below (in the separate progression documents) for all four operations have been taken from the Sense of Number Visual Calculations Policy.

They show, wherever possible, the different strategies for calculation exemplified with identical values. This allows children to compare different strategies and to ask key questions, such as, 'what's the same, what's different?'

M5b: Grid Method
Short Multiplication

$$147 \times 4 = 588$$

x	100	40	7
4	400	160	28

$$400 + 160 + 28 = 588$$

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M6: Expanded Column

$$\begin{array}{r} \begin{array}{ccc} 100 & 10 & 1 \\ 1 & 4 & 7 \\ \times & 4 & \\ \hline 28 & (4 \times 7) \\ 160 & (4 \times 40) \\ 400 & (4 \times 100) \\ \hline 588 \end{array} \end{array}$$

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M7: Column Multiplication

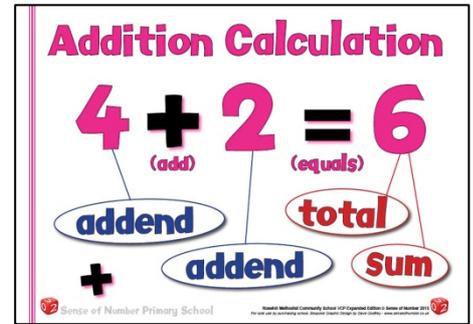
$$\begin{array}{r} \begin{array}{ccc} 100 & 10 & 1 \\ 1 & 4 & 7 \\ \times & 4 & \\ \hline 588 \\ 1 & 2 \end{array} \end{array}$$

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Addition Progression

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method, accurately and with confidence.

Children need to acquire **one efficient written method of calculation** for addition that they know they can rely on **when mental methods are not appropriate**.



To add successfully, children need to be able to:

- Recall all addition pairs to $9 + 9$ and complements in 10.
- Mentally add a series of one-digit numbers, such as $5 + 8 + 4$.
- Add multiples of 10 (such as $60 + 70$) or of 100 (such as $600 + 700$) using the related addition fact, $6 + 7$, and their knowledge of place value.
- Partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

Mental Addition Strategies

There are 5 key mental strategies for addition, which need to be a regular and consistent part of the approach to calculation in all classes from Year 2 upwards.

These strategies will be introduced individually when appropriate, and then be rehearsed and consolidated throughout the year until they are almost second nature.

These strategies are **partitioning, counting on, round and adjust, double and adjust and using number bonds**. The first two strategies are also part of the written calculation policy (see pages 12-14) but can equally be developed as simple mental calculation strategies once children are skilled in using them as jottings.

For example, using the number 45, we can look at the other number chosen, and decide on the most appropriate mental calculation strategy.

<p>MA1: Partitioning</p> $45 + 82 = 127$ $120 + 7 = 127$	<p>MA2: Counting On</p> $45 + 20 = 65$	<p>MA3: Number Bonds</p> $45 + 95 = 140$ $40 + 100 = 140$	<p>MA4: Double & Adjust</p> $45 + 46 = 91$ $45 + 45 + 1 = 91$ $90 + 1 = 91$	<p>MA5: Round & Adjust</p> $45 + 39 = 84$ $45 + 40 - 1 = 84$ $85 - 1 = 84$
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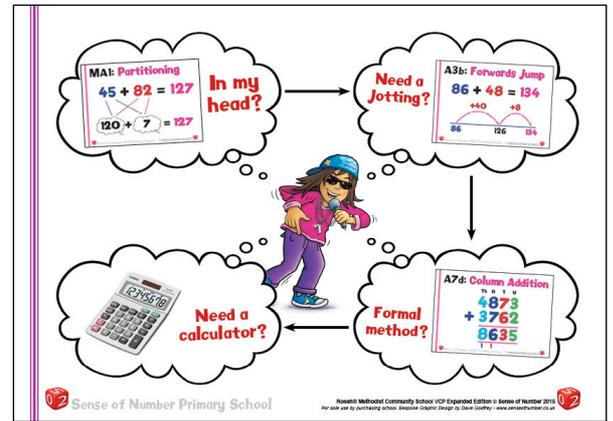
The 5 key strategies need to be linked to the key messages from pages 2 and 3 –

The choice as to whether a child will choose to use a mental method or a jotting will depend upon

- a) the numbers chosen and
- b) the level of maths that the child is working at.

For example, for $57 + 35$

a Year 2 child may use a long jotting or number line
 a Year 3 child might jot down a quick partition jotting,
 a Year 4 child could simply partition and add mentally.

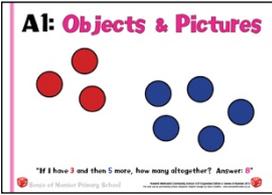
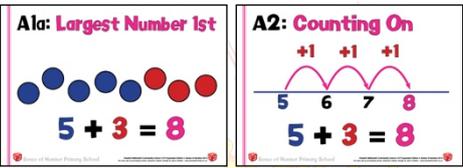
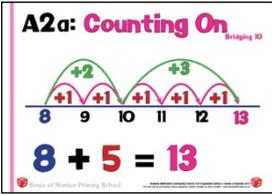
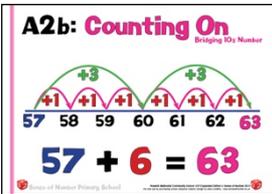


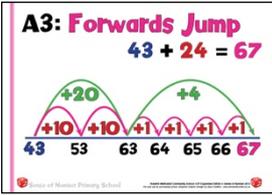
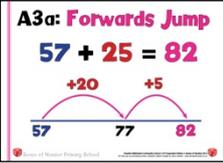
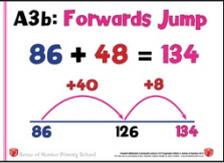
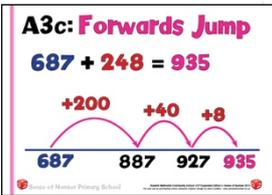
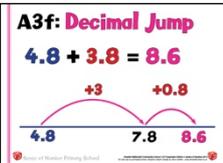
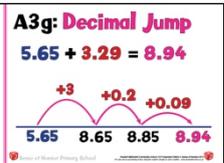
As a strategy develops, a child will begin to recognise the instances when it would be appropriate: -

E.g. $27 + 9$, $434 + 197$, $7.6 + 1.9$ and $5.86 + 3.97$ can all be calculated very quickly by using the **Round & Adjust** strategy.

Below you can see the progression of each strategy through the year groups, with some appropriate examples of numbers, which may be used for each strategy.

MA	MA1: Partitioning $45 + 82 = 127$ $120 + 7 = 127$	MA2: Counting On $45 + 20 = 65$ $45 \xrightarrow{+20} 65$		MA3: Number Bonds $45 + 95 = 140$ $40 + 100 = 140$	MA4: Double & Adjust $45 + 46 = 91$ $45 + 45 + 1 = 91$ $90 + 1 = 91$	MA5: Round & Adjust $45 + 39 = 84$ $45 + 40 - 1 = 84$
Y1		MA2a: Counting On $12 + 5 = 17$ $12 \xrightarrow{+5} 17$	MA2b: Counting On $57 + 10 = 67$ $57 \xrightarrow{+10} 67$	MA3: Number Bonds $3 + 4 + 7 = 14$ $10 + 4 = 14$	MA4: Double & Adjust $5 + 6 = 11$ $5 + 5 + 1 = 11$ $10 + 1 = 11$	MA5: Round & Adjust $45 + 9 = 54$ $45 + 10 - 1 = 54$ $55 - 1 = 54$
Y2	MA1: Partitioning $43 + 21 = 64$ $60 + 4 = 64$	MA2a: Counting On $78 + 7 = 85$ $78 \xrightarrow{+7} 85$	MA2b: Counting On $58 + 40 = 98$ $58 \xrightarrow{+40} 98$	MA3: Number Bonds $3 + 4 + 7 = 14$ $10 + 4 = 14$	MA4: Double & Adjust $7 + 8 = 15$ $7 + 7 + 1 = 15$ $14 + 1 = 15$	MA5: Round & Adjust $45 + 19 = 64$ $45 + 20 - 1 = 64$ $65 - 1 = 64$
Y3	MA1: Partitioning $57 + 25 = 82$ $70 + 12 = 82$	MA2a: Counting On $85 + 50 = 135$ $85 \xrightarrow{+50} 135$	MA2b: Counting On $534 + 300 = 834$ $534 \xrightarrow{+300} 834$	MA3: Number Bonds $43 + 9 + 7 + 21 = 80$ $50 + 30 = 80$	MA4: Double & Adjust $16 + 17 = 33$ $16 + 16 + 1 = 33$ $32 + 1 = 33$	MA5: Round & Adjust $45 + 97 = 142$ $45 + 100 - 3 = 142$ $145 - 3 = 142$
Y4	MA1: Partitioning $648 + 231 = 879$ $800 + 70 + 9 = 879$	MA2a: Counting On $784 + 60 = 844$ $784 \xrightarrow{+60} 844$	MA2b: Counting On $4837 + 3000 = 8347$ $4837 \xrightarrow{+3000} 7837$	MA3: Number Bonds $42 + 16 + 28 + 54 = 140$ $70 + 70 = 140$	MA4: Double & Adjust $37 + 38 = 75$ $37 + 37 + 1 = 75$ $74 + 1 = 75$	MA5: Round & Adjust $345 + 298 = 643$ $345 + 300 - 2 = 643$ $645 - 2 = 643$
Y5	MA1: Partitioning $576 + 258 = 834$ $700 + 120 + 14 = 834$	MA2a: Counting On $837 + 500 = 1337$ $837 \xrightarrow{+500} 1337$	MA2b: Counting On $7583 + 5000 = 12583$ $7583 \xrightarrow{+5000} 12583$	MA3: Number Bonds $£4.56 + £3.27 + £1.44 = £9.27$ $£6.00 + £3.27 = £9.27$	MA4: Double & Adjust $125 + 127 = 252$ $125 + 125 + 2 = 252$ $250 + 2 = 252$	MA5: Round & Adjust $4645 + 1996 = 6641$ $4645 + 2000 - 4 = 6641$ $6645 - 4 = 6641$
Y6	MA1: Partitioning $4.73 + 2.21 = 6.94$ $6 + 0.9 + 0.04 = 6.94$	MA2a: Counting On $43,826 + 30,000 = 73,826$ $43,826 \xrightarrow{+30,000} 73,826$	MA2b: Counting On $5,763,947 + 4,000,000 = 9,763,947$ $5,763,947 \xrightarrow{+4,000,000} 9,763,947$	MA3: Number Bonds $24.25 + 31.63 + 21.75 = 77.63$ $46 + 31.63 = 77.63$	MA4: Double & Adjust $4.5 + 4.7 = 9.2$ $4.5 + 4.5 + 0.2 = 9.2$ $9 + 0.2 = 9.2$	MA5: Round & Adjust $45.2 + 49.9 = 95.1$ $45.2 + 50 - 0.1 = 95.1$ $95.2 - 0.1 = 95.1$

Stage 1	Finding a Total and the Empty Number Line	Alternative Method: Counting on Mentally or as a jotting
FS/Y1	<p>Initially, children need to represent addition using a range of different resources, and understand that a total can be found by counting out the first number, counting out the second number then counting how many there are altogether.</p> <p style="text-align: center;">$3 + 5 = 8$</p>	
		3 (held in head) then use fingers to count on 5 ("3... 4,5,6,7,8")
	<p>This will quickly develop into placing the largest number first, either as a pictorial / visual method or by using a number line.</p> <p style="text-align: center;">$5 + 3 = 8$</p>	
		5 (held in head) then count on 3 ("5 ... 6, 7, 8")
Y1/2	<p>Steps in addition can be recorded on a number line. The steps often bridge through 10.</p> <p style="text-align: center;">$8 + 5 = 13$</p>	
		8 (held in head) then count on 5 ("8 ... 9, 10, 11, 12, 13")
	The next step is to bridge through a multiple of 10.	
		57 (held in head) then count on 6 ("57 ... 58,59,60,61,62,63")
	<p>The number line becomes a key image for demonstrating how to keep one number whole, whilst partitioning the other number.</p> <p>Teach the children firstly to add the tens then the units individually ($43 + 24 = 43 + 10 + 10 + 1 + 1 + 1 + 1$) before progressing to counting on in tens and units ($43 + 20 + 4$)</p>	<p>This method will be a jotting approach, and may look like the following examples: -</p> <p style="text-align: center;">$43 + 24$ $43 + 20 = 63$ $63 + 4 = 67$</p>

		<p>Or</p> $43 + 20 + 4 = 67$
	<p>Develop to crossing the 10s, then the 100s boundary</p> $57 + 25 = 82 \quad 86 + 48 = 134$	
	 	$57 + 25 \qquad 86 + 48$ $57 + 20 = 77 \qquad 86 + 40 = 126$ $77 + 5 = 82 \qquad 126 + 8 = 134$ $57 + 20 + 5 = 82 \qquad 86 + 40 + 8 = 134$
Y3/4	<p>For some children, this method can still be used for 3 digit calculations</p>	<p>Number lines support children's thinking if they find partitioning / column addition difficult, as it simply involves counting on in 100s, 10s & 1s.</p>
		$687 + 248$ $687 + 200 = 887$ $887 + 40 = 927$ $927 + 8 = 935$ <p>Or</p> $687 + 200 + 40 + 8 = 935$
Y5/6	<p>In Years 5 and 6, if necessary, children can return to this method to support their understanding of decimal calculation</p>	
	 	$4.8 + 3.8$ $4.8 + 3 = 7.8$ $7.8 + 0.8 = 8.6$ <p>Or</p> $4.8 + 3 + 0.8 = 8.6$

Number lines support children's thinking if they find partitioning / column addition difficult, as it simply involves counting on in 100s, 10s & 1s.

Hopefully, with the above calculation, many children would mentally Round & Adjust ($4.8 + 4 - 0.2 = 8.6$)

Stage 2	Partition Jot	Alternative Method: Traditional Partitioning
<p>Y2/3</p>	<p>Traditionally, partitioning has been presented using the method on the right. Although this does support place value and the use of arrow cards, it is very laborious, so it is suggested that adopting the 'partition jot' method will improve speed and consistency for mental to written (or written to mental) progression</p>	<p>Record steps in addition using partition, initially as a jotting: -</p> $43 + 24 = 40 + 20 + 3 + 4 =$ $60 + 7 = 67$ <p>Or, preferably</p>
	<p>As soon as possible, refine this method to a much quicker and clearer 'Partition Jot' approach</p> <div data-bbox="564 504 836 694" style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;"> <p>A5: Partition Jot</p> $43 + 24 = 67$ $60 + 7$ </div>	<div data-bbox="1166 450 1394 613" style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;"> <p>A4: Partitioning</p> $43 + 24 = 67$ $40 + 20 = 60$ $3 + 4 = 7$ $\underline{67}$ </div>
	<p>As before, develop these methods, especially Partition Jot, towards crossing the 10s and then 100s.</p>	
	<div data-bbox="467 819 695 981" style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px;"> <p>A5a: Partition Jot</p> $57 + 25 = 82$ $70 + 12$ </div> <div data-bbox="707 819 935 981" style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px;"> <p>A5b: Partition Jot</p> $86 + 48 = 134$ $120 + 14$ </div>	<div data-bbox="1074 819 1273 965" style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px;"> <p>A4b: Partitioning</p> $86 + 48 = 134$ $80 + 40 = 120$ $6 + 8 = 14$ $\underline{134}$ </div> <div data-bbox="1284 819 1484 965" style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px;"> <p>A4a: Partitioning</p> $57 + 25 = 82$ $50 + 20 = 70$ $7 + 5 = 12$ $\underline{82}$ </div>
	<p>This method will soon become the recognised jotting to support the teaching of partitioning. It can be easily extended to 3 and even 4 digit numbers when appropriate.</p>	<p>For certain children, the traditional partitioning method can still be used for 3 digit numbers, but is probably too laborious for 4 digit numbers.</p>
<p>Y3/4</p>	<div data-bbox="426 1169 695 1359" style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px;"> <p>A5c: Partition Jot</p> $687 + 248 = 935$ $800 + 120 + 15$ </div> <div data-bbox="707 1169 976 1359" style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px;"> <p>A5d: Partition Jot</p> $4873 + 3762 = 8635$ $7000 + 1500 + 130 + 5$ </div>	<div data-bbox="1166 1169 1394 1332" style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;"> <p>A4c: Partitioning</p> $687 + 248 = 935$ $600 + 200 = 800$ $80 + 40 = 120$ $7 + 8 = 15$ $\underline{935}$ </div>
	<p>Partition jot is also extremely effective as a quicker alternative to column addition for decimals.</p>	<p>Some simple decimal calculations can also be completed this way.</p>
<p>Y5/6</p>	<div data-bbox="426 1460 695 1650" style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px;"> <p>A5f: Partition Jot</p> $4.8 + 3.8 = 8.6$ $7 + 1.6$ </div> <div data-bbox="707 1460 976 1650" style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px;"> <p>A5g: Partition Jot</p> $5.65 + 3.29 = 8.94$ $8 + 0.8 + 0.14$ </div>	
	<p>For children with higher-level decimal place value skills, partition jot can be used with more complex decimal calculations or money.</p>	
	<div data-bbox="426 1783 695 1973" style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px;"> <p>A5h: Partition Jot</p> $76.7 + 58.5 = 135.2$ $120 + 14 + 1.2$ </div> <div data-bbox="707 1783 976 1973" style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px;"> <p>A5i: Partition Jot</p> $£38.25 + £27.46 = £65.71$ $£65.00 + £0.71$ </div>	

Stage 3	Expanded Method in Columns																															
<p>Y3</p>	<p>Column methods of addition are introduced in Year 3, but it is crucial that they still see mental calculation as their first principle, especially for 2 digit numbers.</p> <p>Column methods should only be used for more difficult calculations, usually with 3 digit numbers that cross the Thousands boundary or most calculations involving 4 digit numbers and above.</p> <p>N.B. Even when dealing with bigger numbers / decimals, children should still look for the opportunity to calculate mentally (E.g. 4675 + 1998)</p>																															
	<p>2 digit examples are used below simply to introduce column methods to the children. Most children would continue to answer these calculations mentally or using a simple jotting.</p>																															
	<p>Using the column, children need to learn the principle of adding the units first rather than the tens.</p> <div data-bbox="810 696 1038 869" style="border: 1px solid black; padding: 5px;"> <p>A6: Expanded Column Addition</p> <p>2 into</p> $24 + 31 = 55$ <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: center;">t</td> <td style="text-align: center;">u</td> </tr> <tr> <td style="text-align: center;">20</td> <td style="text-align: center;">4</td> </tr> <tr> <td style="text-align: center;">30</td> <td style="text-align: center;">1</td> </tr> <tr> <td style="text-align: center;">50</td> <td style="text-align: center;">5</td> </tr> </table> <p><small>© Sense of Number Primary School</small></p> </div>		t	u	20	4	30	1	50	5																						
t	u																															
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	<p>The 'expanded' method is a very effective introduction to column addition. It continues to use the partitioning strategy that the children are already familiar with, but begins to set out calculations vertically. It is particularly helpful for automatically 'dealing' with the 'carry' digit</p>																															
<p>Y3/4</p>	<p>A. Single 'carry' in units</p>	<p>B. 'Carry' in units and tens</p>																														
	<div data-bbox="496 1227 772 1417" style="border: 1px solid black; padding: 5px;"> <p>(A6: Expanded Column) Addition</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: center;">10</td> <td style="text-align: center;">1</td> </tr> <tr> <td style="text-align: center;">57</td> <td></td> </tr> <tr> <td style="text-align: center;">+ 25</td> <td></td> </tr> <tr> <td style="text-align: center;">12</td> <td></td> </tr> <tr> <td style="text-align: center;">70</td> <td></td> </tr> <tr> <td style="text-align: center;">82</td> <td></td> </tr> </table> <p><small>© Sense of Number Primary School</small></p> </div>	10	1	57		+ 25		12		70		82		<div data-bbox="810 1111 1082 1391" style="border: 1px solid black; border-radius: 50%; padding: 10px; color: white; text-align: center;"> <p>'Eighty plus forty equals one hundred and twenty, because 'eight plus four equals twelve.</p> </div> <div data-bbox="1075 1227 1351 1417" style="border: 1px solid black; padding: 5px;"> <p>(A6: Expanded Column) Addition</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: center;">100</td> <td style="text-align: center;">10</td> <td style="text-align: center;">1</td> </tr> <tr> <td style="text-align: center;">86</td> <td></td> <td></td> </tr> <tr> <td style="text-align: center;">+ 48</td> <td></td> <td></td> </tr> <tr> <td style="text-align: center;">14</td> <td></td> <td></td> </tr> <tr> <td style="text-align: center;">120</td> <td></td> <td></td> </tr> <tr> <td style="text-align: center;">134</td> <td></td> <td></td> </tr> </table> <p><small>© Sense of Number Primary School</small></p> </div>	100	10	1	86			+ 48			14			120			134		
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	<p>Once this method is understood, it can quickly be adapted for use with three digit numbers. It is rarely used for 4 digits and beyond as it becomes too inefficient.</p>																															
	<div data-bbox="788 1554 1064 1749" style="border: 1px solid black; padding: 5px;"> <p>A6: Expanded Column Addition</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: center;">100</td> <td style="text-align: center;">10</td> <td style="text-align: center;">1</td> </tr> <tr> <td style="text-align: center;">687</td> <td></td> <td></td> </tr> <tr> <td style="text-align: center;">+ 248</td> <td></td> <td></td> </tr> <tr> <td style="text-align: center;">15</td> <td></td> <td></td> </tr> <tr> <td style="text-align: center;">120</td> <td></td> <td></td> </tr> <tr> <td style="text-align: center;">800</td> <td></td> <td></td> </tr> <tr> <td style="text-align: center;">935</td> <td></td> <td></td> </tr> </table> <p><small>© Sense of Number Primary School</small></p> </div>		100	10	1	687			+ 248			15			120			800			935											
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	<p>The time spent on practising the expanded method will depend on security of number facts recall and understanding of place value.</p> <p>Once the children have had enough experience in using expanded addition, and have also used practical resources (Base 10 / place value counters) to model exchanging in columns, they can be taken on to standard, 'traditional' column addition.</p>																															

Stage 4	Column Method
<p>Y3/4</p>	<p>As with the expanded method, begin with 2 digit numbers, simply to demonstrate the method, before moving to 3 digit numbers.</p> <p>Make it very clear to the children that they are still expected to deal with all 2 digit (and many 3 digit) calculations mentally (or with a jotting), and that the column method is designed for numbers that are too difficult to access using these ways. The column procedure <u>is not</u> intended for use with 2 digit numbers.</p>
	<p>'Carry' units then units and tens</p> <div data-bbox="1066 622 1500 745" style="border: 1px solid black; border-radius: 15px; padding: 5px; display: inline-block; background-color: #f080f0;"> <p>Use the words 'carry ten' and 'carry hundred', not 'carry one'</p> </div>
<div data-bbox="193 779 424 925" style="border: 1px solid black; border-radius: 15px; padding: 5px; display: inline-block; background-color: #f080f0;"> <p>Record carry digits below the line.</p> </div>	<div style="display: flex; justify-content: space-around;"> <div data-bbox="504 757 778 947" style="border: 1px solid black; padding: 5px;"> <p>(A7: Column Addition)</p> $\begin{array}{r} \text{10} \quad \text{1} \\ 57 \\ + 25 \\ \hline 82 \end{array}$ </div> <div data-bbox="786 757 1061 947" style="border: 1px solid black; padding: 5px;"> <p>(A7: Column Addition)</p> $\begin{array}{r} \text{100} \quad \text{10} \quad \text{1} \\ 86 \\ + 48 \\ \hline 134 \end{array}$ </div> <div data-bbox="1069 757 1343 947" style="border: 1px solid black; padding: 5px;"> <p>A7: Column Addition</p> $\begin{array}{r} \text{100} \quad \text{10} \quad \text{1} \\ 687 \\ + 248 \\ \hline 935 \end{array}$ </div> </div>
<p>Y4</p>	<p>Once confident, use with 4 digit numbers (Year 4).</p>
	<div data-bbox="786 1137 1061 1328" style="border: 1px solid black; padding: 5px;"> <p>A7d: Column Addition</p> $\begin{array}{r} 4873 \\ + 3762 \\ \hline 8635 \end{array}$ </div>
<p>Y5/6</p>	<p>Extend to 5/6 digit calculations then decimal calculations (Year 5)</p>
<div data-bbox="151 1570 424 1877" style="border: 1px solid black; border-radius: 15px; padding: 5px; display: inline-block; background-color: #f080f0;"> <p>If children make repeated errors at any stage, they can return to the expanded method or an earlier jotting.</p> </div>	<div style="display: flex; justify-content: space-around;"> <div data-bbox="504 1518 778 1709" style="border: 1px solid black; padding: 5px;"> <p>A7e: Column Addition</p> $\begin{array}{r} 787567 \\ + 446278 \\ \hline 1233845 \end{array}$ </div> <div data-bbox="786 1518 1061 1709" style="border: 1px solid black; padding: 5px;"> <p>A7f: Column Addition</p> $\begin{array}{r} \text{1} \quad \text{1} \quad \text{1} \\ 4.8 \\ + 3.8 \\ \hline 8.6 \end{array}$ </div> <div data-bbox="1069 1518 1343 1709" style="border: 1px solid black; padding: 5px;"> <p>A7g: Column Addition</p> $\begin{array}{r} \text{1} \quad \text{1} \quad \text{1} \\ 5.65 \\ + 3.29 \\ \hline 8.94 \end{array}$ </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div data-bbox="647 1720 922 1910" style="border: 1px solid black; padding: 5px;"> <p>A7h: Column Addition</p> $\begin{array}{r} \text{10} \quad \text{1} \quad \text{1} \\ 76.7 \\ + 58.5 \\ \hline 135.2 \end{array}$ </div> <div data-bbox="930 1720 1204 1910" style="border: 1px solid black; padding: 5px;"> <p>A7i: Column Addition</p> $\begin{array}{r} \text{1} \quad \text{1} \quad \text{1} \\ \text{£}38.25 \\ + \text{£}27.46 \\ \hline \text{£}65.71 \end{array}$ </div> </div>
	<p>The key skill in upper Key Stage 2 that needs to be developed is the laying out of the column method for calculations with decimals in different places.</p>

A7j: Column Addition
With Decimals
 $73.4 + 5.67 = 79.07$

$$\begin{array}{r} 73.4 \\ + 5.67 \\ \hline 79.07 \end{array}$$

Subtraction Progression

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as $160 - 70$) using the related subtraction fact (e.g. $16 - 7$), and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into $70 + 4$ or $60 + 14$).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

Children need to acquire **one efficient written method of calculation for subtraction**, which they know they can rely on **when mental methods are not appropriate**.

NOTE: They should look at the actual numbers each time they see a calculation and decide whether or not their favoured method is most appropriate (e.g. If there are zeroes in a calculation such as $206 - 198$) then the 'counting on' approach may well be the best method in that particular instance).

Therefore, when subtracting, whether mental or written, children will mainly choose between two main strategies to find the difference between two numbers: -

Counting Back (Taking away)

When should we count back and when should we count on?

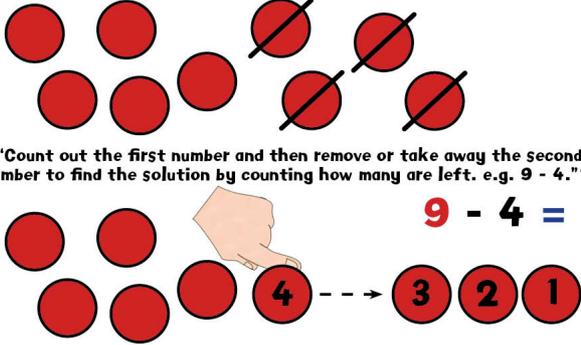
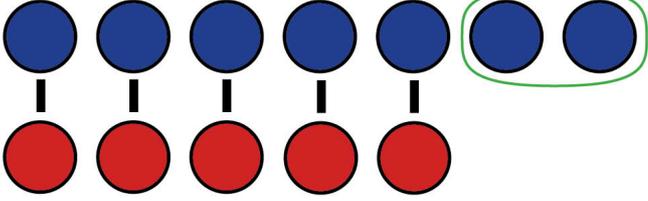
This will alter depending on the calculation (see below), but often the following rules apply;

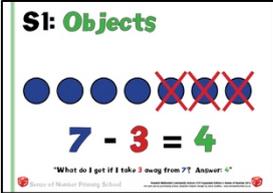
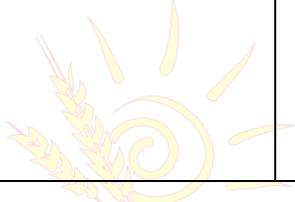
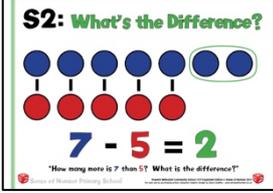
Counting On

If the numbers are far apart, or there isn't much to subtract ($278 - 24$) then count back.

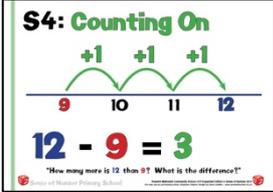
If the numbers are close together ($206 - 188$), then count up

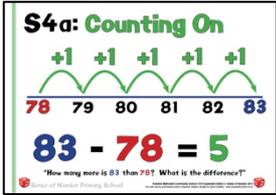
In many cases, either strategy would be suitable, depending on

Models	Subtraction
<p>Removing items from a set:</p> <p>A: Take Away</p> <p>B: Reduction</p> <p>(Count Back Images)</p>	<p>S: Take Way/Reduction</p> <p>Count Back</p>  <p>"Count out the first number and then remove or take away the second number to find the solution by counting how many are left. e.g. $9 - 4 = ?$"</p> <p>$9 - 4 = 5$</p> <p><small>Sense of Number Primary School</small></p>
	<p>Take Away: Samir has 12 cakes and Ben takes 5 cakes. How many cakes does Samir now have?</p> <p>Reduction: The shoes originally cost £12, but have been reduced in the sale by £5. How much do they now cost?</p>
<p>Comparing two sets:</p> <p>A: Comparison</p> <p>B: Inverse of Addition</p> <p>(Counting Up/On Images)</p>	<p>S: Comparison/Inverse of Add</p> <p>Count On</p>  <p>$7 - 5 = 2$</p> <p>"How many more is 7 than 5? What is the difference?"</p> <p><small>Sense of Number Primary School</small></p>
	<p>Comparison: Samir has 12 cakes and Ben has 5 cakes. How many more cakes does Samir have than Ben?</p> <p>Inverse of Addition: The shoes cost £12, but I've only got £5. How much more money will I need in order to buy the shoes? ($5 + ? = 12$)</p>

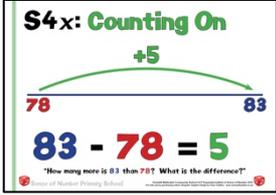
INTRO	Subtraction by counting back (or taking away)	Subtraction by counting up (or complementary addition)
FS/Y1	Early subtraction in EYFS will primarily be concerned with 'taking away' , and will be demonstrated using a wide range of models and resources.	
		
	This will continue in Year 1, using resources and images (including the desktop number track / line) to practise taking away practically, and then counting back on demarcated number lines.	In Year 1, it is also vital that children understand the concept of subtraction as 'finding a difference' and realise that any subtraction can be answered in 2 different ways, either by counting up or counting back. Again, this needs to be modelled and consolidated regularly using a wide range of resources, especially multilink towers, counters and Numicon.
		

Barley Fields

Stage 1	Using the empty number line	
	Subtraction by counting back (or taking away)	Subtraction by counting up (or complementary addition)
	The empty number line helps to record or explain the steps in mental subtraction. It is an ideal model for counting back and bridging ten , as the steps can be shown clearly. It can also show counting up from the smaller to the larger number to find the difference .	
Y1		Small differences can be found by counting up $12 - 9 = 3$
		
Y2/3		For 2 (or 3) digit numbers close together, count up $83 - 78 = 5$ First, count in ones



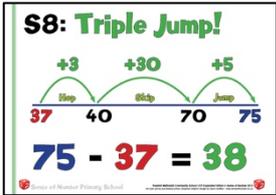
Then use number facts to count in a single jump



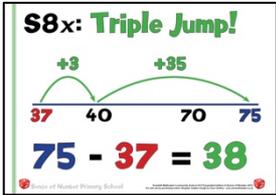
Continue to spot small differences with 3 digit numbers ($403 - 397 = 6$)

Some numbers ($75 - 37$) can be subtracted just as quickly either way.

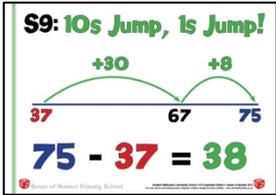
Or count up from smaller to the larger number, initially with a 'triple jump' strategy of jumping to the next 10, then multiples of 10, then to the target number.

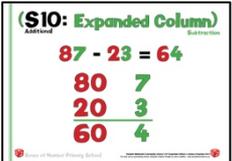
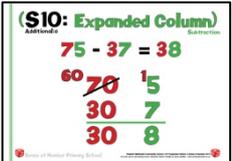
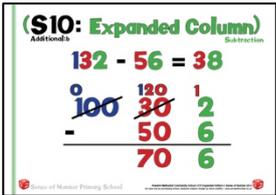
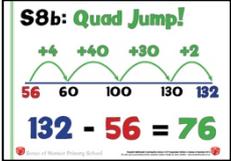
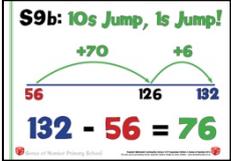


This can also be done in 2 jumps.



Some children prefer to jump in tens and units, which is an equally valid strategy, as it links to the mental skill of 'counting up from any number in tens'



Stage 2	Expanded Method & Number Lines (continued)	
	Subtraction by counting back Expanded Method	Subtraction by counting up Number Lines (continued)
	<p>In Year 3, according to the New Curriculum, children are expected to be able to use both jottings and written column methods to deal with 3 digit subtractions.</p> <p>It is very important that they have had regular opportunities to use the number line 'counting up' approach first (right hand column below) so that they already have a secure method that is almost their first principle for most 2 and 3 digit subtractions.</p> <p>This means that once they have been introduced to the column method they have an alternative approach that is often preferable, depending upon the numbers involved.</p> <p>The number line method also gives those children who can't remember or successfully apply the column method an approach that will work with any numbers (even 4 digit numbers and decimals) if needed.</p> <p>It is advisable in Year 3 to focus upon the number line / counting up approach through regular practice, then introducing column method when appropriate in Year 3.</p> <p>Ideally, whenever columns are introduced, the expanded method should be practised in depth (potentially up until 4 digit calculations are introduced)</p>	
<h1>Y3/4</h1>	<p>The expanded method of subtraction is an excellent way to introduce the column approach as it maintains the place value and is much easier to model practically with place value equipment such as Base 10 or place value counters</p> <p>Introduce the expanded method with 2 digit numbers, but only to explain the process. Column methods are very rarely needed for 2 digit calculations.</p> <p>Partition both numbers into tens and units, firstly with no exchange then exchanging from tens to the units.</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> <p>$87 - 23$</p>  </div> <div style="text-align: center;"> <p>$75 - 37$</p>  </div> </div>	
	<p>Develop into exchanging from hundreds to tens and tens to units.</p> <div style="text-align: center;"> <p>$132 - 56$</p>  </div>	<p>The number line method is equally as effective when crossing the hundreds boundary, either by the triple / quad jump strategy or by counting in tens then units.</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> <p>S8b: Quad Jump!</p>  </div> <div style="text-align: center;"> <p>S9b: 10s Jump, 1s Jump!</p>  </div> </div> <p>The 'quad jump' can be completed by many children in fewer steps, either a triple or double jump.</p>

S8x: Quad Jump!

$132 - 56 = 76$

A

Take the method into three digit numbers.
Subtract the units, then the tens, then the hundreds.
Demonstrate without exchanging first.
 $784 - 351$

For examples without exchanging, the number line method takes considerably longer than mental partitioning or expanded.

S10x: Expanded Column
Subtraction (100, 10, 1)

$$784 - 351 = 433$$

700	80	4
- 300	50	1
400	30	3

B

Move towards exchanging from hundreds to tens and tens to units, in two stages if necessary
 $723 - 356$

S19: Expanded Subtraction

700	50	4
- 200	80	6
400	140	14
- 200	80	6
200	60	8

$754 - 286 = 468$

S10: Expanded Column
Subtraction (100, 10, 1)

$$723 - 356 = 367$$

600	110	1
- 300	50	6
300	60	7

C

Use some examples which include the use of zeros e.g. $605 - 328$.

For numbers containing zeros, counting up is often the most reliable method.

S10: Expanded Column
Subtraction (100, 10, 1)

$$605 - 328 = 277$$

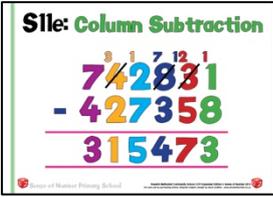
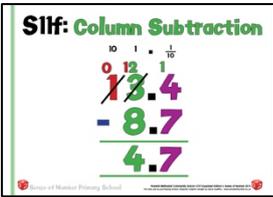
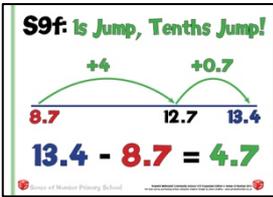
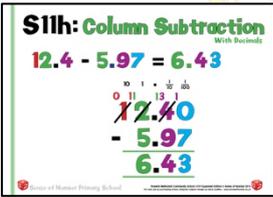
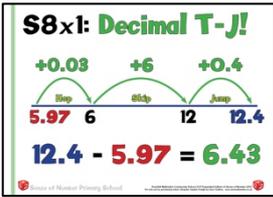
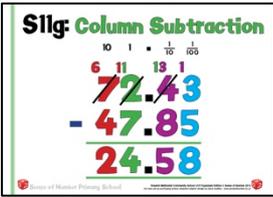
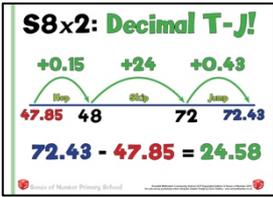
500	90	1
- 300	20	6
300	70	7

S4x: Counting On

$605 - 328 = 277$

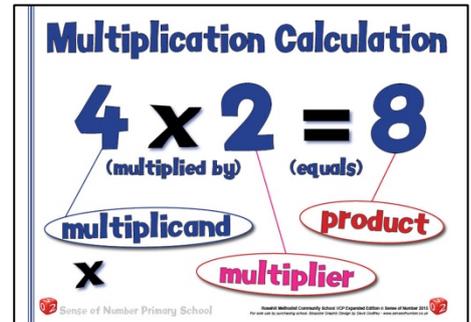
Continue to use expanded subtraction until both number facts and place value are considered to be very secure!

Stage 3	Standard Column Method (decomposition)	
	Subtraction by counting back Standard Method	Subtraction by counting up Number Lines (continued)
Mainly Y4+	<p>Decomposition relies on secure understanding of the expanded method, and simply displays the same numbers in a contracted form.</p> <p>As with expanded method, and using practical resources such as place value counters to support the teaching, children in Years 3 or 4 (depending when the school introduces the column procedure) will quickly move from decomposition via 2-digit number 'starter' examples to 2 / 3 digit and then 3 digit columns.</p> <p style="text-align: center;">75 – 37 132 – 56</p> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 5px;"> <p>(S1): Column Subtraction</p> $\begin{array}{r} \text{10} \quad \text{1} \\ \text{6} \quad \text{7} \quad \text{5} \\ - \quad \text{3} \quad \text{7} \\ \hline \text{3} \quad \text{8} \end{array}$ </div> <div style="border: 1px solid black; padding: 5px;"> <p>(S1): Column Subtraction</p> $\begin{array}{r} \text{100} \quad \text{10} \quad \text{1} \\ \text{0} \quad \text{1} \quad \text{2} \quad \text{1} \\ \text{1} \quad \text{3} \quad \text{2} \\ - \quad \text{5} \quad \text{6} \\ \hline \text{7} \quad \text{6} \end{array}$ </div> </div>	
	<p style="text-align: center;">723 – 356</p> <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; padding: 5px; margin-right: 10px;"> <p>S1: Column Subtraction</p> $\begin{array}{r} \text{100} \quad \text{10} \quad \text{1} \\ \text{6} \quad \text{2} \quad \text{3} \\ - \quad \text{3} \quad \text{5} \quad \text{6} \\ \hline \text{3} \quad \text{6} \quad \text{7} \end{array}$ </div> <div style="border: 2px solid purple; border-radius: 15px; padding: 10px; background-color: #e0ffe0;"> <p><i>Continue to refer to digits by their actual value, not their digit value, when explaining a calculation. E.g. One hundred and twenty subtract fifty.</i></p> </div> </div>	
	<p>Again, use examples containing zeros, remembering that it may be easier to count on with these numbers (see Stage 2)</p> <p style="text-align: center;">605 – 328</p> <div style="border: 1px solid black; padding: 5px; margin: 0 auto; width: fit-content;"> <p>S1x: Column Subtraction</p> $\begin{array}{r} \text{100} \quad \text{10} \quad \text{1} \\ \text{5} \quad \text{0} \quad \text{5} \\ - \quad \text{3} \quad \text{2} \quad \text{8} \\ \hline \text{3} \quad \text{6} \quad \text{7} \end{array}$ </div>	
Y4	<p>From late Y4 onwards, move onto examples using 4 digit (or larger) numbers and then onto decimal calculations.</p> <p style="text-align: center;">5042 – 1776</p> <div style="border: 1px solid black; padding: 5px; margin: 0 auto; width: fit-content;"> <p>S1d: Column Subtraction</p> $\begin{array}{r} \text{4} \quad \text{0} \quad \text{10} \quad \text{1} \\ \text{5} \quad \text{0} \quad \text{4} \quad \text{2} \\ - \quad \text{1} \quad \text{7} \quad \text{7} \quad \text{6} \\ \hline \text{3} \quad \text{2} \quad \text{6} \quad \text{6} \end{array}$ </div>	

<h1>Y5/6</h1>	<p>In Years 5 & 6 apply to any 'big number' examples.</p>	
	 <p>S1e: Column Subtraction</p> $\begin{array}{r} 742831 \\ - 427358 \\ \hline 315473 \end{array}$	
	<p>Both methods can be used with decimals, although the counting up method becomes less efficient and reliable when calculating with more than two decimal places.</p>	
	<p>13.4 – 8.7</p>	<p>13.4 – 8.7</p>
	 <p>S1f: Column Subtraction</p> $\begin{array}{r} 13.4 \\ - 8.7 \\ \hline 4.7 \end{array}$	 <p>S9f: Is Jump, Tenths Jump!</p> $\begin{array}{c} +4 \qquad +0.7 \\ \text{-----} \\ 8.7 \qquad 12.7 \qquad 13.4 \\ 13.4 - 8.7 = 4.7 \end{array}$
	<p>12.4 – 5.97</p>	<p>12.4 – 5.97</p>
	 <p>S11h: Column Subtraction With Tenths</p> $\begin{array}{r} 12.40 \\ - 5.97 \\ \hline 6.43 \end{array}$	 <p>S8x1: Decimal T-J!</p> $\begin{array}{c} +0.03 \quad +6 \quad +0.4 \\ \text{-----} \\ 5.97 \quad 6 \quad 12 \quad 12.4 \\ 12.4 - 5.97 = 6.43 \end{array}$
	<p>72.43 – 47.85</p>	
	 <p>S11g: Column Subtraction</p> $\begin{array}{r} 72.43 \\ - 47.85 \\ \hline 24.58 \end{array}$	 <p>S8x2: Decimal T-J!</p> $\begin{array}{c} +0.15 \quad +24 \quad +0.43 \\ \text{-----} \\ 47.85 \quad 48 \quad 72 \quad 72.43 \\ 72.43 - 47.85 = 24.58 \end{array}$

Multiplication Progression

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.



These notes show the stages in building up to using an efficient method for

- **two-digit by one-digit** multiplication by the end of **Year 3**,
- **three-digit by one-digit** multiplication by the end of **Year 4**,
- **four-digit by one-digit** multiplication **and two/three-digit by two-digit** multiplication by the end of **Year 5**
- **three/four-digit by two-digit** multiplication **and** multiplying **1-digit numbers with up to 2 decimal places by whole numbers** by the end of **Year 6**.

To multiply successfully, children need to be able to:

- **recall all multiplication facts to 12×12** ;
- **partition numbers into multiples of one hundred, ten and one**;
- **work out products such as 70×5 , 70×50 , 700×5 or 700×50 using the related fact 7×5 and their knowledge of place value**;
- **similarly apply their knowledge to simple decimal multiplications such as 0.7×5 , 0.7×0.5 , 7×0.05 , 0.7×50 using the related fact 7×5 and their knowledge of place value**;
- **add two or more single-digit numbers mentally**;
- **add multiples of 10 (such as $60 + 70$) or of 100 (such as $600 + 700$) using the related addition fact, $6 + 7$, and their knowledge of place value**;
- **add combinations of whole numbers using the column method (see above)**.

Note:

Children need to acquire **one efficient written method of calculation for multiplication, which they know they can rely on when mental methods are not appropriate.**

It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

These mental methods are often more efficient than written methods when multiplying.

Use partitioning and grid methods until number facts and place value are secure

For a calculation such as 25×24 , a quicker method would be 'there are four 25s in 100 so $25 \times 24 = 100 \times 6 = 600$ '

When multiplying a 3 / 4 digit x 2-digit number the standard method is usually the most efficient

**At all stages, use known facts to find other facts.
E.g. Find 7×8 by using 5×8 (40) and 2×8 (16)**

Mental Multiplication

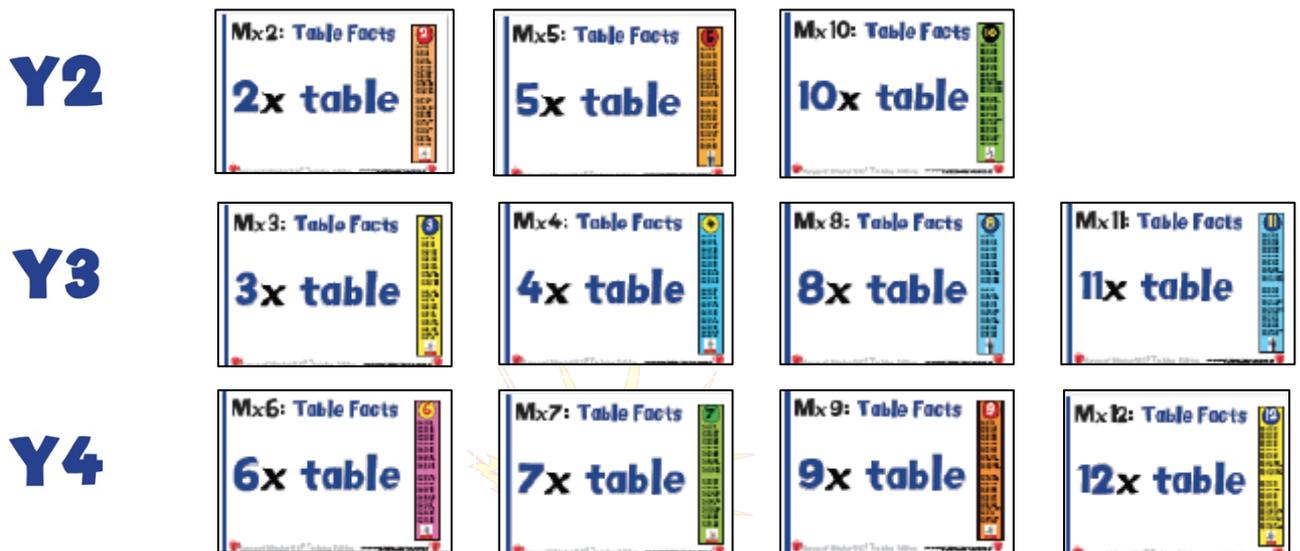
In a similar way to addition, multiplication has a range of mental strategies that need to be developed both before and then alongside written methods (both informal and formal).

Tables Facts

In Key Stage 2, however, before any written methods can be securely understood, children need to have a bank of multiplication tables facts at their disposal, which can be recalled instantly.

The learning of tables facts does begin with counting up in different steps, but by the end of Year 4 it is expected that most children can instantly recall all facts up to 12 x 12.

The progression in facts is as follows (11's moved into Y3 as it is a much easier table to recall): -



Once the children have established a bank of facts, they are ready to be introduced to jottings and eventually written methods.

Doubles & Halves

The other facts that children need to know by heart are doubles and halves. If children haven't learned to recall simple doubles instantly, and haven't been taught strategies for mental doubling, then they cannot access many of the mental calculation strategies for multiplication (E.g. Double the 4 times table to get the 8 times table. Double again for the 16 times table etc.).

As a general guidance, children should know the following doubles: -

Year Group	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Doubles and Halves	All doubles and halves from double 1 to double 10 / half of 2 to half of 20	All doubles and halves from double 1 to double 20 / half of 2 to half of 40 (E.g. double 17=34, half of 28 = 14)	Doubles of all numbers to 100 with units digits 5 or less, and corresponding halves (E.g. Double 43, double 72, half of 46) Reinforce doubles & halves of all multiples of 10 & 100 (E.g. double 800, half of 140)	Addition doubles of numbers 1 to 100 (E.g. 38 + 38, 76 + 76) and their corresponding halves Revise doubles of multiples of 10 and 100 and corresponding halves	Doubles and halves of decimals to 10 – 1 d.p. (E.g. double 3.4, half of 5.6)	Doubles and halves of decimals to 100 – 2 d.p. (E.g. double 18.45, half of 6.48)

Before certain doubles / halves can be recalled, children can use a simple jotting to help them record their steps towards working out a double / half.

Y2

MM5: Doubling
Double 17 = 34
 $20 + 14 = 34$

MM5a: Doubling
Double 37 = 74
 $60 + 14 = 74$

Y3

MM5b: Doubling
Double 78 = 156
 $140 + 16 = 156$

MM5c: Doubling
Double 340 = 680
 $600 + 80 = 680$

MM5d: Doubling
Double 480 = 960
 $800 + 160 = 960$

Y4

MM5e: Doubling
Double 278 = 556
 $400 + 140 + 16 = 556$

Y4/5

MM5f: Doubling
Double 768 = 1536
 $1400 + 120 + 16 = 1536$

MM5g: Doubling
Double 3.7 = 7.4
 $6 + 1.4 = 7.4$

As mentioned, though, there are also several mental calculation strategies that need to be taught so that children can continue to begin any calculation with the question 'Can I do it in my head?' The majority of these strategies are usually taught in Years 4 – 6, but there is no reason why some of them cannot be taught earlier as part of the basic rules of mathematics.

Partitioning is an equally valuable strategy for multiplication, and can be quickly developed from a jotting to a method completed entirely mentally. It is clearly linked to the grid method of multiplication, but should also be taught as a 'partition jot' so that children, by the end of Year 4, have become skilled in mentally partitioning 2 and 3 digit numbers when multiplying (with jottings when needed).

By the time they leave Year 6 they should be able to mentally partition most simple 2 & 3 digit, and also decimal multiplications.

MM3: Partitioning
 $15 \times 5 = 75$
 $50 + 25 = 75$
 $(10 \times 5) \quad (5 \times 5)$

MM3a: Partitioning
 $37 \times 4 = 148$
 $120 + 28 = 148$
 $(30 \times 4) \quad (7 \times 4)$

Round & Adjust is also a high quality mental strategy for multiplication, especially when dealing with money problems in upper KS2. Once children are totally secure with rounding and adjusting in addition, they can be shown how the strategy extends into multiplication, where they round then adjust by the multiplier.

E.g. For 39×6 round to 40×6 (240) then adjust by 1×6 (6) to give a product of $240 - 6 = 234$.

MM4: Round & Adjust

$$49 \times 3 = 147$$

$$(50 \times 3) - (1 \times 3)$$

$$150 - 3 = 147$$

MM4a: Round & Adjust

$$198 \times 4 = 792$$

$$(200 \times 4) - (2 \times 4)$$

$$800 - 8 = 792$$

MM4b: Round & Adjust

$$3.9 \times 5 = 19.5$$

$$(4 \times 5) - (0.1 \times 5)$$

$$20 - 0.5 = 19.5$$

MM4c: Round & Adjust

$$£5.99 \times 6 = £35.94$$

$$(£6 \times 6) - (1p \times 6)$$

$$£36 - 6p = £35.94$$

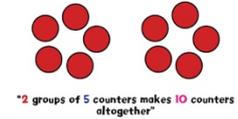
Y4

Y4/5

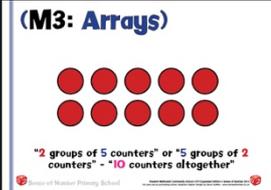
Y5

Y5/6

Written Multiplication

Stage 1	Number Lines, Arrays & Mental Methods
<p>FS</p>	<p>In Early Years, children are introduced to grouping, and are given regular opportunities to put objects into groups of 2, 3, 4, 5 and 10. They also stand in different sized groups, and use the term 'pairs' to represent groups of 2.</p> <p>Using resources such as Base 10 apparatus, Numicon, multi-link or counters, children visualise counting in ones, twos, fives and tens, saying the multiples as they count the pieces. E.g. Saying '10, 20, 30' or 'Ten, 2 tens, 3 tens' whilst counting Base 10 pieces</p>
<p>Y1</p>	<p>Begin by introducing the concept of multiplication as repeated addition.</p> <p>Children will make and draw objects in groups (again using resources such as Numicon, counters and multi-link), giving the product by counting up in 2s, 5s, 10s and beyond, and writing the multiplication statement.</p> <div data-bbox="820 1529 1093 1722" style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p>(M1: Groups)</p>  </div> <p>Extend into making multiplication statements for 3s and 4s, using resources (especially real life equipment such as cups, cakes, sweets etc.)</p> <p>Make sure from the start that all children say the multiplication fact the correct way round, using the word 'multiply' more often than the word 'times'.</p> <p>For the example above, there are 5 counters in 2 groups, showing 5 multiplied by 2 (5x2), not 2 times 5. It is the '5' which is being scaled up / made bigger / multiplied.</p> <p>'5 multiplied by 2' shows '2 groups of 5' or 'Two fives'</p>

Develop the use of the array to show linked facts (repeated addition).
Emphasise that all multiplications can be worked out either way. ($2 \times 5 = 5 \times 2 = 10$)



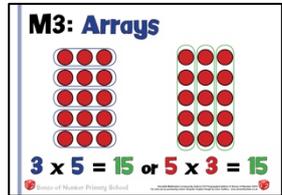
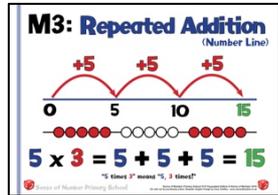
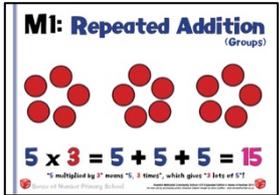
Y2

Build on children's understanding that multiplication is repeated addition, using arrays and number lines to support their thinking. Explore arrays in real life.



Start to emphasise commutativity, e.g. that $5 \times 3 = 3 \times 5$

*Continue to emphasise multiplication the correct way round.
E.g. $5 \times 3 = 5 + 5 + 5$
 5 multiplied by $3 = 15$*

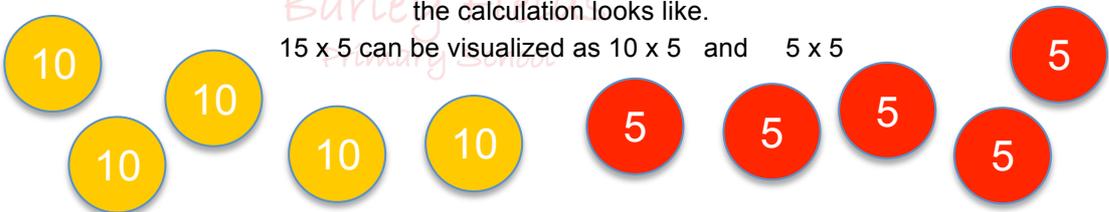


Y3

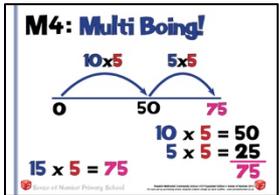
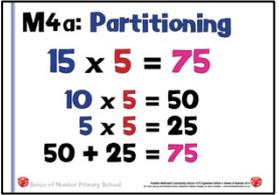
Extend the above methods to include the 3, 4, 6 and 8 times tables.
Continue to model calculations, where appropriate, with resources such as Numicon, Place Value Counters, counting quickly in different steps and placing / moving the resource.

Extend the use of resources to 2 digit x 1 digit calculations so that children can visualize what the calculation looks like.

15×5 can be visualized as 10×5 and 5×5



Then begin to partition using **jottings and number lines**.



Each of these methods can be used in the future if children find expanded or standard methods difficult.

Extend the methods above to calculations which give products greater than 100.

NB. – Use of ‘grid’ method within the New Curriculum

Barley Fields Primary school multiplication grid method is a key element of this policy, but, to align with the New Curriculum, could be classed as a mental ‘jotting’ as it builds on partitioning, and is also the key mental multiplication method used by children in KS2 (see page 29 – multiplication partitioning).

<p>M5: Grid Method Short Multiplication</p> $15 \times 5 = 75$	<p>M5a: Grid Method Short Multiplication</p> $43 \times 6 = 258$	<p>M5b: Grid Method Short Multiplication</p> $147 \times 4 = 588$	<p>M8: Grid Method Long Multiplication</p> $43 \times 65 = 2795$
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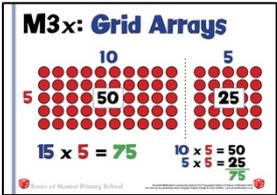
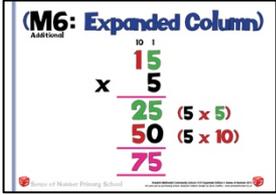
Column procedures still retain some element of place value, but, particularly for long multiplication, tend to rely on memorising a ‘method’ and can lead to many children making errors with the method (which order to multiply the digits, when to ‘add the zero’, dealing with the ‘carry’ digits’ etc.) rather than the actual calculation.

M9: Long Multiplication
Column

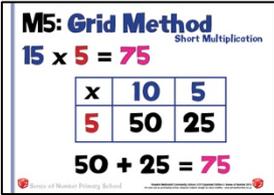
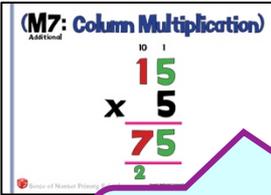
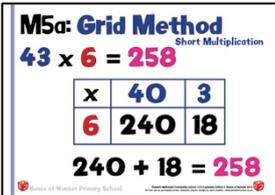
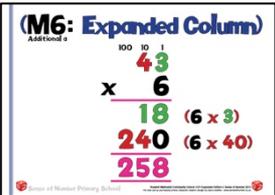
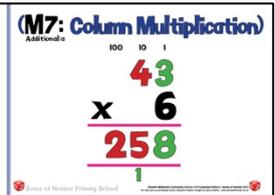
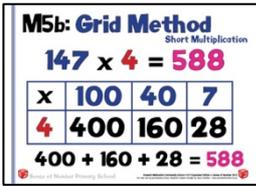
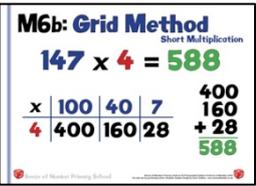
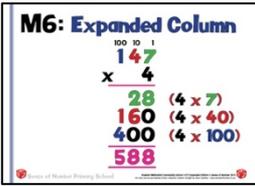
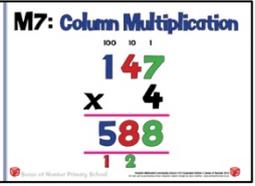
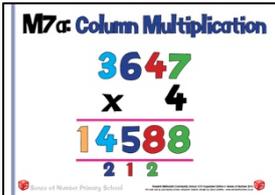
$$\begin{array}{r} 43 \\ \times 65 \\ \hline 215 \quad (5 \times 43) \\ + 2580 \quad (60 \times 43) \\ \hline 2795 \end{array}$$

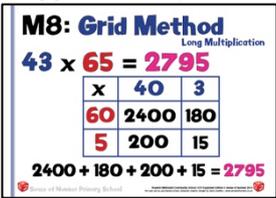
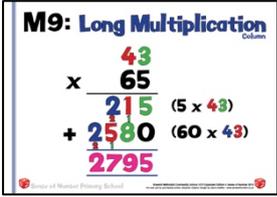
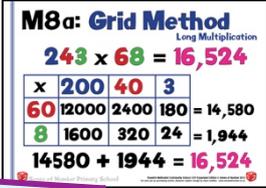
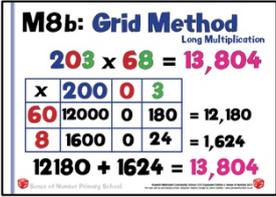
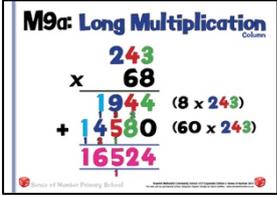
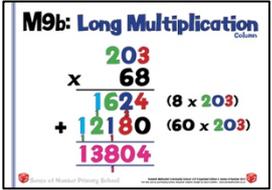
Once the calculations become more unwieldy (4 digit x 1 digit or 3 / 4 digit x 2 digit) then grid method begins to lose its effectiveness, as there are too many zeroes and part products to deal with. At this stage column procedures are far easier, and, once learned, can be applied much more quickly. Grid methods can still be used by some pupils who find columns difficult to remember, and who regularly make errors, but children should be encouraged to move towards columns for more complex calculations.

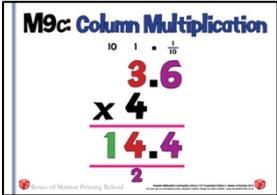
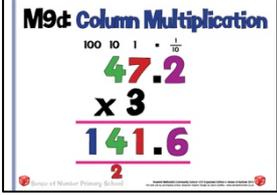
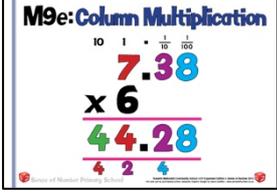
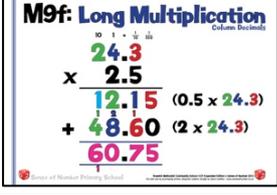
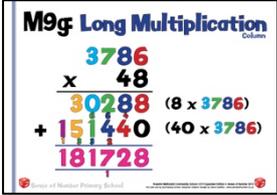
<p>M8a: Grid Method Long Multiplication</p> $243 \times 68 = 16,524$	<p>M9a: Long Multiplication Column</p> $\begin{array}{r} 243 \\ \times 68 \\ \hline 1944 \quad (8 \times 243) \\ + 14580 \quad (60 \times 243) \\ \hline 16524 \end{array}$
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Stage 2	Written Methods - Short Multiplication	
	Grid Multiplication (Mental ‘Jotting’)	Column multiplication (Expanded method into standard)
	<p>The grid method of multiplication is a simple, alternative way of recording the jottings shown previously.</p> <p><i>If necessary (for some children) it can initially be taught using an array to show the actual product.</i></p>	<p>The expanded method links the grid method to the standard method.</p> <p>It still relies on partitioning the tens and units, but sets out the products vertically.</p> <p>Children will use the expanded method until they can securely use and explain the standard method.</p>
		
Y3	<p>It is recommended that the grid method is used as the main method within Year 3. It clearly maintains place value, and helps children to visualise and understand the calculation better.</p>	<p>At some point within the year, the column method can be introduced, and children given the choice of using either grid or standard.</p>

Place the ‘carry’ digit below the line

		 <p style="border: 2px solid purple; border-radius: 15px; padding: 10px; display: inline-block; margin-top: 10px;">When setting out calculations vertically, begin with the ones first (as with addition and subtraction).</p>		
<h1 style="color: blue;">Y4</h1>	<p>Continue to use both grid and column methods in Year 4 for more difficult 2 digit x 1 digit calculations, extending the use of the grid method into mental partitioning for those children who can use the method this way.</p> <p>At this point, the expanded method can still be used when necessary (to help 'bridge' grid with column), but children should be encouraged to use their favoured method (grid or column) whenever possible.</p>			
				
	<p>For 3 digit x 1 digit calculations, both grid and standard methods are efficient. Continue to use the grid method to aid place value and mental arithmetic. Develop column method for speed, and to make the transition to long multiplication easier. If both methods are taught consistently, then children in Year 4 will have a clear choice of 2 secure methods, and will be able to develop both accuracy and speed in multiplication.</p> <div style="border: 2px solid purple; border-radius: 15px; padding: 10px; display: inline-block; margin-top: 10px;"> <p><i>If children find it difficult to add the 'part products' then set them out vertically (or encourage column method)</i></p> </div>			
				
<h1 style="color: blue;">Y5</h1>	<p>For a 4 digit x 1 digit calculation, the column method, once mastered, is quicker and less prone to error. The grid method may continue to be the main method used by children who find it difficult to remember the column procedure, or children who need the visual link to place value.</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;">  </div>			

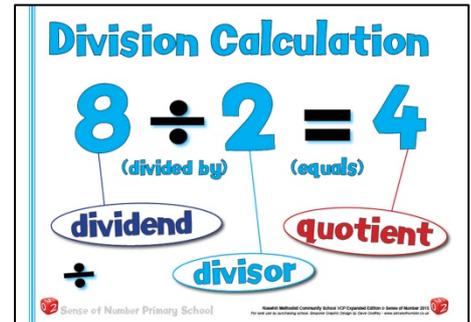
Stage 3	Long Multiplication (TU x TU)	
	Grid Multiplication	Column multiplication (Expanded method into standard)
Y5	<p>Extend the grid method to TU × TU, asking children to estimate first so that they have a general idea of the answer. (43×65 is approximately $40 \times 70 = 2800$.)</p>  <p>As mentioned earlier, the grid method is often the 'choice' of many children in Years 5 and 6, due to its ease in both procedure and understanding / place value and is the method that they will mainly use for simple long multiplication calculations.</p>	<p>Children should only use the 'standard' column method of long multiplication if they can regularly get the correct answer using this method.</p>  <p>There is no 'rule' regarding the position of the 'carry' digits. Each choice has advantages and complications. Either carry the digits mentally or have your own favoured position for these digits.</p>
Y6	<p>For 3 digit x 2 digit calculations, grid method is quite inefficient, and has much scope for error due to the number of 'part-products' that need to be added.</p> <p>Use this method when you find the standard method to be unreliable or difficult to remember.</p>	<p>Most children, at this point, should be encouraged to choose the standard method. For 3 digit x 2 digit calculations it is especially efficient, and less prone to errors when mastered. Although they may find the grid method easier to apply, it is much slower / less efficient.</p>
	<p>Again, estimate first: 243×68 is approximately $200 \times 70 = 14000$.</p>	
	 <p>Add these numbers for the overall product</p> 	
		
	<p>Many children will find the use of Grid method as an efficient method for multiplying decimals.</p>	<p>Extend the use of standard method into the use of decimals.</p>

		
Y6		
		
		
		<p>By this time children meet 4 digits by 2 digits, the only efficient method is the standard method for Long Multiplication.</p>
		

Division Progression

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

These notes show the stages in building up to long division through Years 3 to 6 – first using short division 2 digits \div 1 digit, extending to 3 / 4 digits \div 1 digit, then long division 4 / 5 digits \div 2 digits.



To divide successfully in their heads, children need to be able to:

- **Understand and use the vocabulary of division** – for example in $18 \div 3 = 6$, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient.
- **Partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.**
- **Recall multiplication and division facts to 12×12 , recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value.**
- **Know how to find a remainder working mentally** – for example, find the remainder when 48 is divided by 5.
- **Understand and use multiplication and division as inverse operations.**

Children need to acquire **one efficient written method of calculation for division**, which they know they can rely on **when mental methods are not appropriate**.

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

To carry out expanded and standard written methods of division successful, children also need to be able to:

- **Visualise how to calculate the quotient by visualising repeated addition.**
- **Estimate how many times one number divides into another** – for example, approximately how many sixes there are in 99, or how many 23s there are in 100.
- **Multiply a two-digit number by a single-digit number mentally.**
- **Understand and use the relationship between single digit multiplication, and multiplying by a multiple of 10.** (e.g. $4 \times 7 = 28$ so $4 \times 70 = 280$ or $40 \times 7 = 280$ or $4 \times 700 = 2800$).
- **Subtract numbers using the column method (if using NNS 'chunking').**

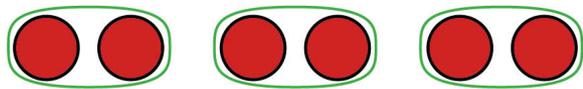
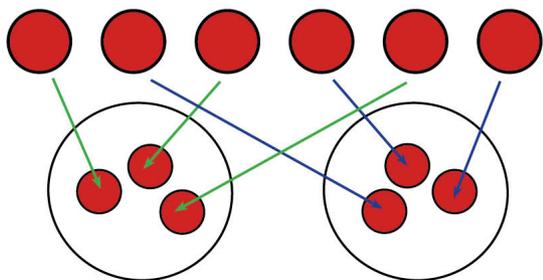
For example, without a clear understanding that 72 can be partitioned into 60 and 12, 40 and 32 or 30 and 42 (as well as 70 and 2), it would be difficult to divide 72 by 6, 4 or 3 using the 'chunking' method.

$72 \div 6$ 'chunks' into 60 and 12

$72 \div 4$ 'chunks' into 40 and 32

$72 \div 3$ 'chunks' into 30 and 42 (or 30, 30 and 12)

The above points are crucial. If children do not have a secure understanding of these prior-learning objectives then they are unlikely to divide with confidence or success, especially when attempting the 'chunking' method of division.

Models	Division
<p style="text-align: center;">Grouping (The key model for division)</p>	<p style="text-align: center;">D: Grouping</p>  <p style="text-align: center;">“How many groups of 2 can I make out of 6?” Answer: 3”</p> <p style="font-size: small; text-align: center;">Sense of Number Primary School Rosehill Methodist Community School VCP Expanded Edition © Sense of Number 2015 For sale use by purchasing school. Bespoke Graphics Design by Clare Godfrey - www.senseofnumber.co.uk</p>
<p style="text-align: center;">Sharing (The model that links with fractions)</p>	<p style="text-align: center;">D: Sharing</p>  <p style="text-align: center;">“If I share 6 into 2 equal amounts, how many in each group?” Answer: 3</p> <p style="font-size: small; text-align: center;">Sense of Number Primary School Rosehill Methodist Community School VCP Expanded Edition © Sense of Number 2015 For sale use by purchasing school. Bespoke Graphics Design by Clare Godfrey - www.senseofnumber.co.uk</p>



Division In Key Stage 1 – Grouping or Sharing?

When children think conceptually about division, their default understanding should be that 'Division is Grouping', as this is the most efficient way to divide.

The 'traditional' approach to the introduction of division in KS1 is to begin with 'sharing', as this is seen to be more 'natural' and easier to understand.

Most children then spend the majority of their time 'sharing' counters and other resources (i.e. seeing $20 \div 5$ as 20 shared between 5).

These children are given little opportunity to use the grouping approach.

(i.e. $20 \div 5$ means how many 5's are there in 20?) – far simpler and can quickly be achieved by counting in 5s to 20, something which most children in Y1 can do relatively easily.

Grouping in division can also be visualised extremely effectively using number lines and Numicon. The only way to visualise sharing is through counting.

Grouping, not sharing, is the inverse of multiplication.

Sharing is division as fractions.

Once children have grouping as their first principle for division they can answer any simple calculation by counting in different steps (2s, 5s, 10s then 3s, 4s, 6s etc.). As soon as they learn their tables facts then they can answer immediately.

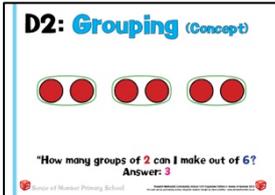
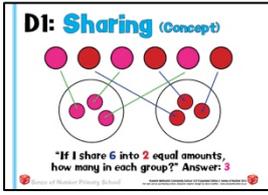
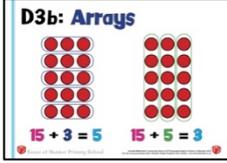
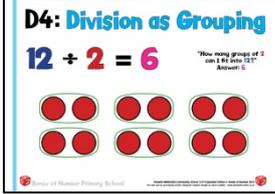
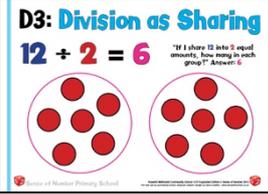
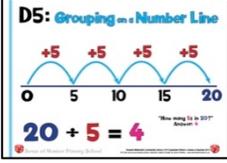
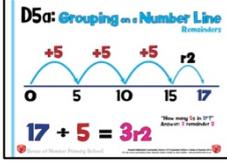
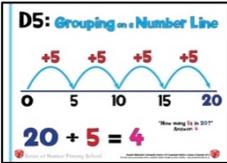
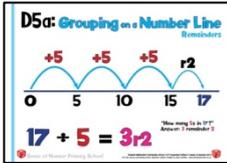
E.g. How much quicker can a child answer the calculations $24 \div 2$, $35 \div 5$ or $70 \div 10$ using grouping? Children taught sharing would find it very difficult to even attempt these calculations.

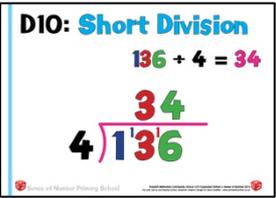
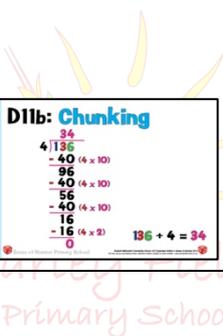
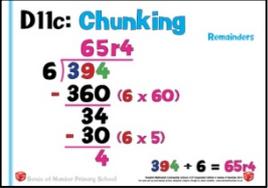
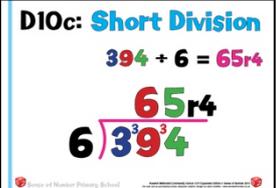
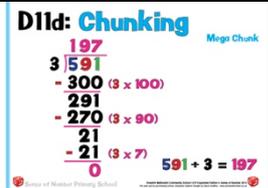
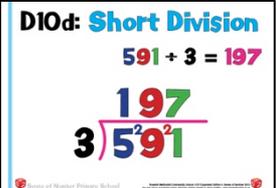
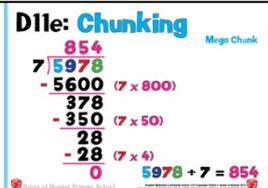
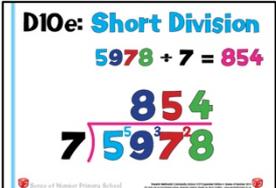
When children are taught grouping as their default method for simple division questions it means that they;

- Secure understanding that the divisor is crucially important in the calculation
- Understand the link to counting in equal steps on a number line
- Have images to support understanding of what to do with remainders (Numicon)
- Have a far more efficient method as the divisor increases
- Have a much firmer basis on which to build KS2 division strategies

Consequently this policy is structured around the teaching of division as grouping, moving from counting up in different steps to learning tables facts and eventually progressing towards the mental chunking, short and long division methods of written division in KS2.

Sharing is introduced as division in KS1, but is then taught mainly as part of the fractions curriculum, where the link between fractions and division is emphasised and maintained throughout KS2.

Stage 1	Concepts and Number Lines (pre-chunking)	
	Grouping	Sharing
FS	From EYFS onwards, children need to explore practically both grouping and sharing . Links can then be made in both KS1 and KS2 between sharing and fractions.	
Y1	Begin by giving children opportunities to use concrete objects, pictorial representations and arrays with the support of the teacher. Use the words 'sharing' and 'grouping' to identify the concepts involved. Identify the link between multiplication and division using the array image.	
		
	 	
Y2	Identify Grouping as the key model for division. Relate to knowledge of multiplication facts. Use the key vocabulary: '20 ÷ 5 means how many 5's can I fit into 20?'	Identify Sharing as the secondary model of division.
		
	Counting on is the easiest route when using a number line to solve a division calculation.	
	 	
Y3	Continue to give children practical images for division by grouping: e.g. use PE mats and ask children to move into groups of 4. The remainder go into a hoop. Use Numicon shapes – how many 4 pieces can I fit into 27 (made of two tens and a seven piece).	<div style="border: 2px solid purple; border-radius: 20px; padding: 10px; width: fit-content; margin: auto;"> <p>Regularly stress the link between multiplication and division, and how children can use their tables facts to divide by counting forwards in steps.</p> </div>
	 	

Stage 2	Chunking & Standard Methods	
	Chunking	Standard Methods
	As previously encountered in Y2, developing an understanding of division with the number line is an excellent way of linking division to multiplication. Division can show both as repeated subtraction, but it is simpler to show division by counting forward to find how many times one number 'goes into' another.	
Y3		
Y4	Begin with the short division method but if appropriate move onto chunking.	
Y5		
		
		
		

Begin by subtracting several chunks, but then try to find the biggest chunks of the divisor that can be

Children should develop the ability to represent the quotient to include a straight forward remainder, but also as a decimal or fractional remainder.

D11f: Chunking Mega Chunk

$$\begin{array}{r} 169r1 \\ 5 \overline{)846} \\ - 500 \quad (5 \times 100) \\ \hline 346 \\ - 300 \quad (5 \times 60) \\ \hline 46 \\ - 45 \quad (5 \times 9) \\ \hline 1 \end{array} \quad 846 \div 5 = 169r1$$

D10f: Short Division Different Remainders

$$\begin{array}{r} 169.2 \\ 5 \overline{)846.0} \end{array} \quad 846 \div 5$$

$$\begin{array}{r} 169r1 \\ 5 \overline{)846} \end{array} \quad 5 \overline{)169\frac{1}{5}}$$

Y6

When introducing long division, it is often easier to find the quotient using the Mega Hunk strategy.

D11g1: Chunking Long Division

$$\begin{array}{r} 32 \\ 15 \overline{)480} \\ - 450 \quad (15 \times 30) \\ \hline 30 \\ - 30 \quad (15 \times 2) \\ \hline 0 \end{array} \quad 480 \div 15 = 32$$

D11g2: Chunking Long Division

$$\begin{array}{r} 32 \\ 15 \overline{)480} \\ - 150 \quad (15 \times 10) \\ \hline 330 \\ - 150 \quad (15 \times 10) \\ \hline 180 \\ - 150 \quad (15 \times 10) \\ \hline 30 \\ - 30 \quad (15 \times 2) \\ \hline 0 \end{array} \quad 480 \div 15 = 32$$

D10i: Short Division

$$87.5 \div 7 = 12.5$$

$$\begin{array}{r} 12.5 \\ 7 \overline{)87.5} \end{array}$$

There are three different ways of calculating using Long Division: the Short Division method, the Traditional method and the NNS Chunking method. The Traditional Long Division method ignores place value, and therefore is not as helpful as the Chunking method, which now becomes the recommended strategy.

D13: Long Division Chunking Method

$$\begin{array}{r} 26r21 \\ 37 \overline{)983} \\ - 740 \quad (37 \times 20) \\ \hline 243 \\ - 222 \quad (37 \times 6) \\ \hline 21 \end{array} \quad 983 \div 37 = 26r21$$

D13j: Long Division Chunking Method

$$\begin{array}{r} 26r21 \\ 37 \overline{)983} \\ - 370 \quad (37 \times 10) \\ \hline 613 \\ - 370 \quad (37 \times 10) \\ \hline 243 \\ - 222 \quad (37 \times 6) \\ \hline 21 \end{array} \quad 983 \div 37 = 26r21$$

D12: Long Division Short Division Method

$$37 \overline{)983} \quad 26r21$$

D14: Long Division Traditional Method

$$\begin{array}{r} 26r21 \\ 37 \overline{)983} \\ - 74 \\ \hline 243 \\ - 222 \\ \hline 21 \end{array} \quad 983 \div 37 = 26r21$$